

Solve Absolute Value Equations

$$a|bx+c|+d=e$$

Key Idea: Isolate the absolute value part of the equation.

$$a|bx+c|+d=e$$

$$a|bx+c|=e-d$$

$$|bx+c|=\frac{e-d}{a}$$

$$\odot |bx+c|=f$$

Case 1: $f < 0$

No solution

Case 2: $f = 0$

1 solution, 1 equation

$$bx+c=0$$

Case 3: $f > 0$

2 solutions, 2 equations

$$bx+c=f \text{ or } bx+c=-f$$

Ex 1: $|x|=3$ [Case III]

$$x=3 \text{ or } x=-3$$

$$\boxed{x=-3, 3}$$

or

$$\boxed{x=\pm 3}$$

Ex 2: $|x-1|=3$ [Case III]

$$x-1=3 \text{ or } x-1=-3$$

$$x=4 \text{ or } x=-2$$

$$\boxed{x=-2, 4}$$

$ -2-1 $	$ 4-1 $
$ -3 $	$ 3 $
$3 \checkmark$	$3 \checkmark$

$$\text{Ex 3: } \frac{2|x|}{2} = \frac{8}{2}$$

$$|x| = 4 \quad \boxed{\text{III}}$$

$$x = 4 \text{ or } x = -4$$

$$\boxed{x = \pm 4}$$

$$\text{Ex 4: } \frac{2|x+1|}{2} = \frac{8}{2}$$

$$|x+1| = 4 \quad \boxed{\text{III}}$$

$$x+1 = 4 \text{ or } x+1 = -4$$

$$x = 3 \text{ or } x = -5$$

$$\boxed{x = -5, 3}$$

$$\begin{array}{cc} 2|-5+1| & 2|3+1| \\ 2|-4| & 2|4| \\ 8\checkmark & 8\checkmark \end{array}$$

$$\text{Ex 5: } \frac{3|x+8|}{-1} + \frac{1}{-1} = 1$$

$$\frac{3|x+8|}{3} = \frac{0}{3}$$

$$|x+8| = 0 \quad \boxed{\text{Case II}}$$

$$x+8 = 0$$

$$\boxed{x = -8}$$

$$3|-8+8| + 1$$

$$3|0| + 1$$

$$1\checkmark$$

$$\text{Ex 6: } \frac{1}{2}|2x+3| + \frac{5}{-5} = \frac{8}{-5}$$

$$(2) \frac{1}{2}|2x+3| = 3 \quad (2)$$

$$|2x+3| = 6 \quad \boxed{\text{III}}$$

$$2x+3 = 6 \text{ or } 2x+3 = -6$$

$$2x = 3$$

$$2x = -9$$

$$x = \frac{3}{2} \text{ or } x = -\frac{9}{2}$$

$$\boxed{x = -\frac{9}{2}, \frac{3}{2}}$$

$$\frac{1}{2}(6) + 5$$

$$3 + 5$$

$$8\checkmark$$

$$\text{Ex 7: } 5|3x-1|+1=1$$

$$5|3x-1|=0$$

$$|3x-1|=0$$

Case II

$$3x-1=0$$

$$3x=1$$

$$x=\frac{1}{3}$$

$$5|1-1|+1$$

$$5|0|+1$$

1 ✓

$$\text{Ex 8: } \frac{5}{8}|x-1|+7=2$$

$$\left(\frac{8}{5}\right)\frac{5}{8}|x-1|=-5\left(\frac{8}{5}\right)$$

$$|x-1|=-8$$

Case I

No solution

Assignment 32:

Part I: p. 393 #15-32

Part II: Solving Absolute Value Equations

Solutions to the Solving Absolute Value Equations Packet

1. $x = -7, -1$

2. $x = -3, 15$

3. $x = -20, -4$

4. $x = -10, 4$

5. $x = 0, 20$

6. $x = \frac{1}{2}, \frac{7}{2}$

7. $x = -8, -2$

8. $x = -30, 21$

9. $x = -6, 4$

10. $x = 3, 11$

11. $x = -5, \frac{13}{3}$

12. $x = -\frac{11}{5}, 3$

13. $x = 2, \frac{8}{3}$

14. $x = -\frac{7}{2}, -2$

15. $x = -3$

16. No solution